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Review

5-axis flank milling: A state-of-the-art review

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ABSTRACT

Flank milling is of importance to machining aircraft structural parts, turbines, blades and several other mechanical parts. It decreases manufacturing time, enhances quality and reduces cost. Since flank milling developable ruled surfaces do not contain geometrical errors, research on flank milling focuses on the generation of optimal tool trajectory for non-developable ruled surfaces, even generic free-form surfaces. This includes: envelope surfaces, geometrical errors (overcut, undercut), energy optimization in tool movement, surface deviations, tool geometry adaptation, tool wear and temperature, and surface roughness. In this article we present a survey on flank milling as well as suggesting guidelines for future considerations in solving flank milling tool trajectory optimization.

Contents

1. Introduction

Flank milling is of importance to machine complex surfaces found mainly in aircraft structural parts, turbines, and blades [1]. Tönshoff et al. [2] highlights the importance of flank milling manufacturing time, quality enhancement and cost reduction. It is mainly applicable to ruled surfaces [3], a particular geometrical type of surface obtained by the motion of a rule on guiding rails. It should also be developable [4], where surface normals along the same rule are collinear, even though few attempts have been made to override the constraints [5]. This survey of the literature recalls the advancement in the field of flank milling research as well as laying the foundation for future work on solving flank milling tool trajectory optimization, or at least defining the important parameters affecting the quality of the finished surface. Research on

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flank milling can be split into two main categories: Optimal tool trajectories and tool geometry.

The article first presents a Preamble, then in Section 2.1, manufacturing strategies as well as an introduction to the mathematics of ruled surfaces. The preamble also includes some definitions that might be useful in consolidating the terms used throughout the various reviewed publications. Following on from this, we attempt to review the literature with respect to the earlier defined two main categories (see Fig. 1). Section 3 investigates category 1: the generation of the optimal tool trajectory through error reduction. This can be divided into the following:

- Calculation of errors, the ability to rectify the tool path of twisted surfaces to obtain a machined surface faithful to the intended design.
- Minimization of pure geometrical errors such as overcut (over-indentation of the tool in the surface) and undercut (falling short of removing all excess material).
- Identification of the envelope surface, sometimes referred to as machining surface, serving as tool trajectory guides. It is often seen as a transformation of the original surface along the local normals.
- Prediction of surface roughness and comparison of the tolerance with acceptable design limits.

Section 4 explores category 2: tool adaptation and suitability for the manufacturing mode at hand. This branch can mainly be split into three different problems:

- Study of surface deviations due to tool deflections. In fact, tools are subject to higher flank forces leading to potentially considerable tool deflection affecting the final quality of the machined surface. An algorithm not accounting for potential tool deflection will present sizable differences between the simulation and the real values. The difference is proportional to the rule length.
- Usage of different tool geometries to overcome the manufacturing problem. Traditionally a cylindrical flat tool is used, but recently conical tools as well as different geometries have been used.
- Regarding tool geometry, some have considered adapting the tool geometry, as in manufacturing special tools to obtain preset manufacturing surfaces.
- Global tool collision with the surrounding environment—other than the surface under investigation.
- Minimization of tool energy expenditures by carefully observing that algorithms do not propose sudden tool direction changes to suit geometrical error optimization.

Section 5 explores additional research relevant to the topic:

- Assessing tool flank wear, as well as the analysis of tool temperatures in flank milling and its effect on surface quality.
- Sweep milling—the alternative when flank milling is not applicable. The latter investigation exposes new ideas that can be translated and adapted within flank milling.

The article ends with a summary of the literature review findings and with a discussion of future directions to undertake in resolving flank milling tool trajectory computation accounting for realistic flank force deflections.

2. Preamble

In this section we introduce the topic parameters and restrictions, develop an understanding of the nature of the problem, as well as its applicability limitation. We also propose a set of definitions that would serve to standardize the terms used over the diverse reviewed publications.

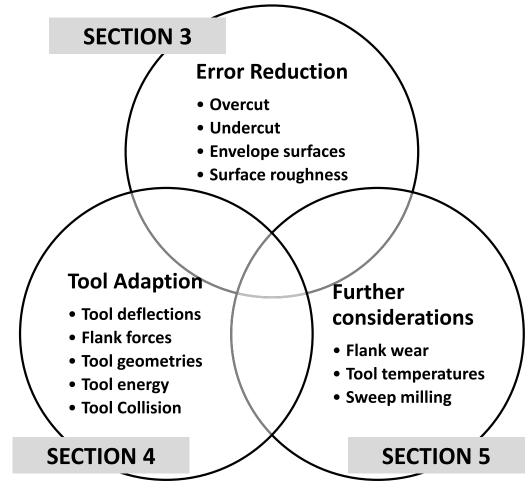

Fig. 1. Flank milling literature review and article sections.

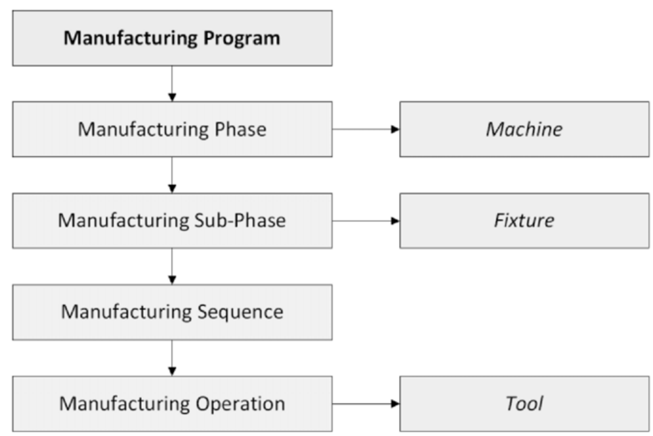

Fig. 2. Manufacturing program subdivision.

2.1. Manufacturing strategies

Using computer numerically controlled machinery (CNC) to manufacture a part requires the setup of the manufacturing program (Fig. 2). It requires the identification of the optimal machinery, fixture selection, tooling (Fig. 4) and manufacturing modes. The latter requires the identification of the tool positions and orientations with regard to the machined surface.

The manufacturing mode of interest in our analysis is flank milling. Manufacturing modes (Fig. 3) can be classified into:

- Flank milling (sometimes referred to as side or peripheral milling): chips formed along the flank side of the tool.
- End milling: particular sweep milling where the contact point is the midst of the tool flat end.

2.2. Ruled surfaces

Ruled surfaces are the results of the movement of a line along guiding curves. This section presents the mathematics of ruled surfaces required for the following paragraphs. For a detailed insight on ruled surfaces, readers can refer to [6,7,3].

A ruled surface is generated by joining corresponding points on two guiding rails – $S(u)$ and $T(u)$ – linearly. The parametric equation of the ruled surface given in Fig. 5 is:

$$P(u, v) = (1 - v)\boldsymbol{S}(u) + v\boldsymbol{T}(u) \quad u \in [0; 1] \;\&\; v \in [0; 1]. \qquad (1)$$

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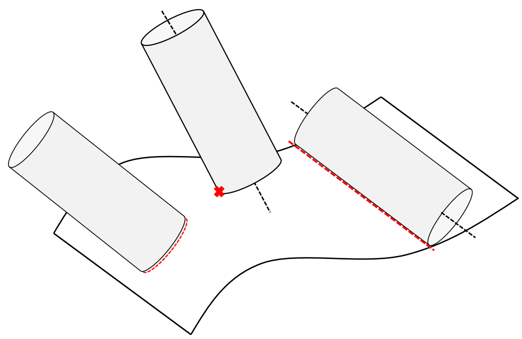

Fig. 3. (From left to right) End milling, sweep milling and flank milling; red areas depict cutter contact locations with surface. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

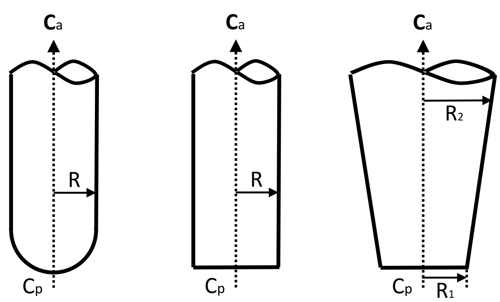

Fig. 4. (a) Ball nose, (b) flat-end mill cylindrical and (c) conical tools.

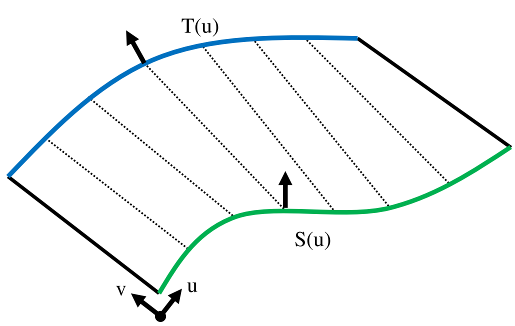

Fig. 5. A ruled surface.

The study of the surface normals along the rules identifies whether the surface is also developable. Normals are computed as follows:

$$\mathbf{N}(u, v) = \frac{\partial \mathbf{P}}{\partial u} \times \frac{\partial \mathbf{P}}{\partial v}. \quad (2)$$

A developable ruled surface has its normals collinear along the same rule (Fig. 6). The developability of a surface is indicated through the parameter 'twisted'. When literature refers to 'twisted ruled surface' it means a non developable ruled surface. The twistability degree is the angular difference between surface normals at both ends of a single rule. It is worthwhile mentioning that planar, cylindrical and conical surfaces are particular developable ruled surfaces. More generally, the envelope surface of a family of planes is also a developable ruled surface.

2.3. Definitions

In this section we present and pictorially define parameters of interest: designed surface, tool path, envelope surface, constraint surface, machined surface, local overcut, global overcut, and undercut. The above parameters are linked each other and most

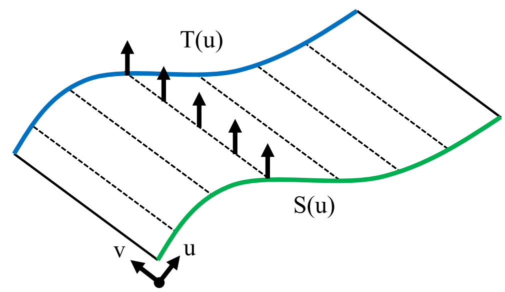

Fig. 6. A developable ruled surface with collinear normals along the same rule.

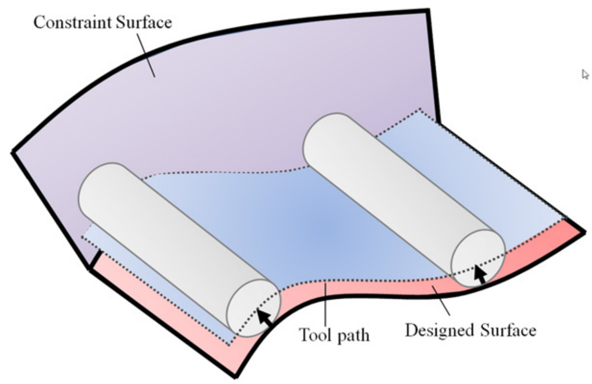

Fig. 7. Designed, envelope and constraint surfaces.

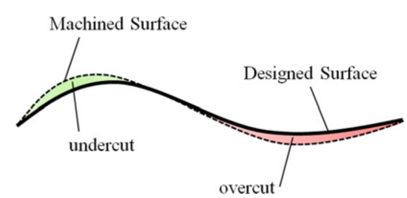

Fig. 8. Undercut and overcut (amplified for representation). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

of the time the computation of one requires the identification of another. We refer to billet as the original volume of material. The term billet is similar to stock, workpart and workpiece. We refer to part as the finished product. A manufacturing program (Fig. 2) is applied to obtain a part out of a billet. We refer to 'designed surface' (Fig. 7) as the surface undergoing process planning. It is the surface that we wish to compute the optimal tool path for its manufacturing. The 'constraint surfaces' are other surfaces that prevent the tool trajectory from obtaining the 'designed surface'.

The 'envelope surface' is the boundary surface formed by the tool path trajectory. It results from the tool sweeping. The 'machined surface' is the actually obtained surface post-manufacture. The differences between the computed surface and the machined surface are related to many factors, such as tool deflection, tool wear, kinematics, feed drives, errors associated to the non-ideal machine tool and so on. The aim is to generate a machined surface as close as possible to the designed surface.

For clarity we will refer to the difference α as the difference between the designed surface and the envelope surface; and to the difference β as the difference between the envelope surface and the machined surface. Local overcut is the amount of material taken out – that should not have been removed – at a specific tool position. The global overcut (red on Fig. 8) is the union of local overcuts across the toolpath. The term undercut (Fig. 8) represents the excess material remaining after the application of the manufacturing process. This excess can only be computed

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Table 1
Comparison between end and flank milling.

Mode	Advantages	Disadvantages
End	– Suitable for large sculpture surfaces – Uses flat end cutter	– Lesser amount of manufacturing
Flank	– Uses cylindrical cutters – Suitable for small and medium surfaces	– Difficult gouge avoidance – Complex tool interference – More tool strength

globally and should be finished using a different process at a second stage.

3. Tool trajectory optimization

Tool trajectory optimization attempts to compensate the errors introduced during 5-axis machining (undercut and overcut). Bohez [8] presents an extensive literature review for error types and their compensation. The literature review has its roots in [9], where errors in modeling and machining of centrifugal impellers are studied. The authors state that literature (prior to 2000) neglected modeling errors (α) and restricted itself to measurement and compensation (β). It served in studying 5-axis machining tool kinematic chain design and analysis [10,11]. Also, tool collision and interference [12,13] is scarce in the field of 5-axis flank milling. Several references can be found for the problem in end/sweep milling [14,15], but only a couple of Refs. [16–18] are found dealing with constraints while attempting to flank mill ruled surfaces. The constraints are not internal (i.e., existence of a boss in the midst of a surface); they are, rather, boundary limitations. In [18] the objective is to improve the kinematic behavior of machine tools during milling. In the following, we will first attempt to chronologically review flank milling literature dealing with tool trajectory optimization. The review will present the main concept and results. For detailed application of the algorithm, the reader is encouraged to consult the reference. We will then present a summary map dynamically linking the studied research.

3.1. Chronological review of tool trajectory optimization related to 5-axis flank milling

One of the first publications dedicated to error computation in flank milling [19] (article in German) describes the economic importance of flank milling and discusses methods of reducing surface discrepancies without providing a detailed error computation algorithm. The topic seems to have been attempted in German industries as [20] presents an initial theoretical model to compute flank milling based on another German reference. The first article to detail flank milling error computation is [21]. It offers a comparison between end [22] and flank milling detailed in Table 1. The main contribution is in the algorithms SPO (single point offset) and DPO (double point offset). They calculated the cutter location data (referred to as CLDATA) using vectors as presented in [23].

Algorithms are described analytically and experimented with on a test surface. In SPO (Fig. 9), the tool is identified by the position of tool center (O) and tool direction (T_a). T_a is collinear to the surface tangent b at P, while O is vectorially computed. Similarly the DPO method is applied on the same test surface. Two points are selected along the rules (in a uniform subdivision of the rule). These points are offset at a distance equal to the tool radius along the surface normal. The vector joining these two points forms the proposed orientation of the tool.

Wu [24] demonstrates that a surface (complex arbitrary surfaces) can be flank millable as well as ruled surfaces. The author emphasizes the importance of flank milling, or else the slow machining process of point milling should be employed. The work conducted at Pratt & Whitney Canada Inc. presented the desire to

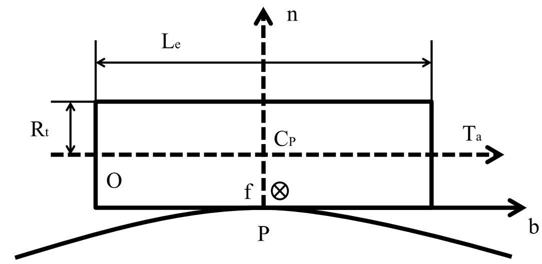


Fig. 9. SPO method of [21].

expand the domain of applicability of flank milling to cover axial compressors in the form of integrally bladed rotors. By imposing three design curves to lie on a highly twisted ruled surface, and through reducing the deviation between the ruled surface and the machined surface by introducing the technique of multiple pass flank milling, they were able to flank mill two moderately complex rotors. Elber and Fish [25] studies the ability to flank mill surfaces by approximating any general surface into a ruled one. The target is the machining of saddle-like (hyperbolic) and convex surfaces rather than minimizing errors. The authors do not show an interest in exploring time optimization. Attempting to simultaneously remove higher volumes of material is not investigated. Redonnet et al. [26] proposes to 'share' the error between the two optimal tool positions of non-developable ruled surfaces. The optimal positions were previously defined as placing the tool tangent at the guiding rails referencing [27]. This proposition is based on adding a third tangency condition: Tangency of the tool lower generating line to the rule (besides the tangency to the two guiding rails on both sides of the extreme points of the rule). A proposed method was applied on Table 10. A quantitative analysis comparing [21–26], gave an error of 0.22 mm using the same tool dimensions (whereas it was 0.585 mm in [21]). Abdel-Malek and Yeh [28] shows a methodology to compute tool swept volume that can be used to approximate errors. Leu et al. [29] argues that previous methodologies are not suitable for NC real time data machining verification and presents, as an alternative, the sweep-envelope differential equation.

Rubio et al. [27] developed an overall 5-axis machining program for free-form surface. Of the several proposed algorithms, flank milling trajectories were investigated. The tool is positioned collinear to the rule considered. Two positions are investigated: placing the tool, respectively, tangent to each guiding rail. Error calculations generate a three-equation system with four unknowns; a tolerance is then selected to resolve the system. An example is carried on a test figure (surface data not given). Monies et al. [30–32], Redonnet et al. [26], Senatore et al. [5,33,34] are considered as a continuum to the effort initiated by Rubio et al. [27]. Monies et al. [30] suggests an improved positioning tending towards achieving two goals: interference and process time reduction. The tool positioning is based on having one point of contact (M_2 in Fig. 10) and a two points tangency (M_0 and M_1). The milling cutter is rotated around y_2. The algorithm generates seven equations with seven unknowns (the positioning parameters) and is solved based on a Newton method. Errors are computed on surface, defined in Table 10, and are compared to [27]. Results show

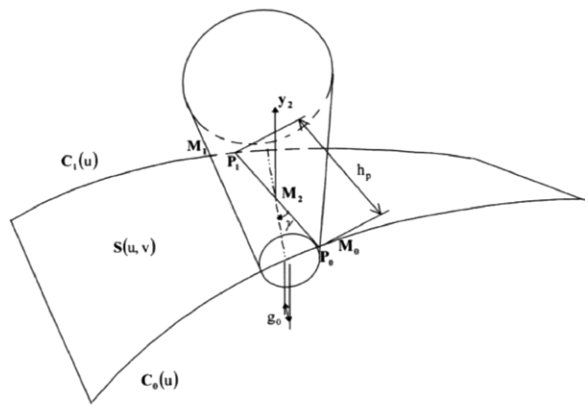


Fig. 10. Positioning algorithm of [30].

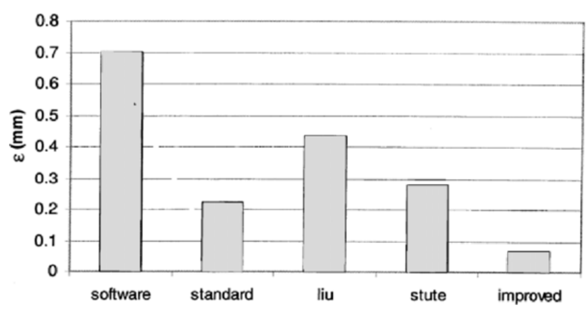


Fig. 11. Maximum errors of different algorithms according to [31].

drastic improvements of errors, mainly as the minimum radius at the end of the conical milling cutter increases. In [31] the authors compare the algorithms of [19,21,27–30], while presenting in depth analysis for all of the respective algorithms (see Fig. 11). Lin and Koren [35] developed a feasibility study for an 5-axis machining interpolator, assuming parts are gouge-free.

Tsay and Her [36] establishes an analytical model to reduce discrepancies between the designed and machined twisted ruled surfaces. The author uses the term twist to identify the more recognized term of 'non-developable'. The error is computed based on a series of projections and geometrical analysis. The author concludes that undercutting is automatically related to the 'twisting angle', that is, the degree of non-developability. Several numerical examples are given studying the variation of several parameters, as well as a test case. A comparison of average errors is computed and presented. The methodology is used as a base, where the authors present in [37] a flank cutting technology that can be applied to generate 5-axis cutter paths for machining centrifugal compressor impellers. Although [38] uses the ruled surfaces theory to generate a toolpath, the application is not directed at manufacturing a ruled surface, but rather on describing a tool path benefiting from ruled surfaces curvature properties. The real-time control approach uses high-order motion properties and is suitable for high-precision machining of free-formed surfaces. Tournier et al. [39] and Duc et al. [40] puts forward the concept of the machining surface. In this paper, a first machining surface is calculated representing the tool trajectory (set of tool axes in flank milling). This machining surface is described as a ruled surface, the guiding curves of which correspond to two *B*-spline curves. The envelope surface of the tool movement during milling is approximated, following a kinematical approach, with the aim of calculating geometrical errors. Therefore, the control points of the *B*-spline curves (standing for the machining surface) are moved so that the geometrical deviations are minimized. The concept was enlarged in [41] by the introduction of energy optimization to preserve the smoothness of the trajectory.

Table 2
Comparison of results between [21,46] (units are in mm).

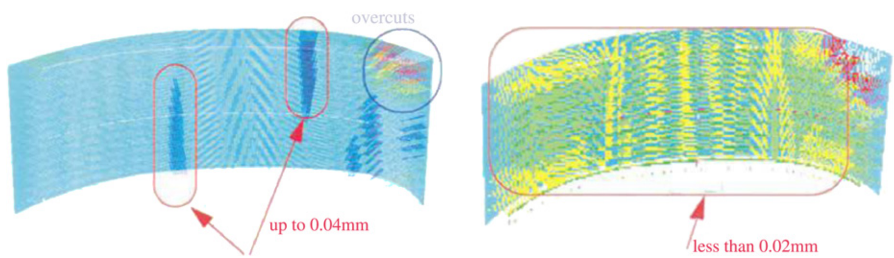
	[21]	[46]
Maximum undercut	0.582	0.2644
Maximum overcut	0.585	0.2114
Maximum angle change	–	0.6426

This work is completed in [42]. The machining surface concept was also presented in [39]. It was developed using end milling with several tool geometries. It was not implemented for flank milling, but the proposition is highly valuable and can be applied to flank milling. Lartigue et al. [43] introduced an assessment and correction methodology for the tool path while performing a 5-axis flank milling operation. At first the tool path is simulated using the envelope surface. A kinematic approach is used and presented for the particular case of flank milling. The appendix provides a detailed calculation that leads to a set of curves, later interpolated to obtain the envelope surface. At a second stage, geometrical deviations between the cutting tool and the designed surface to be machined are calculated. Following this, a correction of the tool path is proposed by displacing points of the envelope surface accounting for the error in geometrical deviations. The proposition is verified on a sample ruled surface whose parameters are not given. The error evaluation is measured to read up to 0.04 mm and, when corrected, they reached less than 0.02 mm. This method did not mention how to generate an effective initial toolpath (see Fig. 12).

Bedi et al. [44] opens a series of tool trajectory planning initiatives [45,46]. The strategy places the tool tangential to the top and bottom curves at equal values of u (the non-ruled direction). The methodology consists of building the Frenet–Serret frame at the rule extremities. Then the tool axis is found through the identification of two points on the (T, N) Frenet plane. The points are an intersection result between the tangential positioning of the tool at the guiding curves and the (N, B) planes respectively. Further to this proposition, and through a series of geometric algebraic computations on vectors, the authors obtain a system of four equations with four unknowns. The latter is solved through numerical computations. The error deviation is then approximated at the mid-curve of the ruled surfaces (see Table 11). Menzel et al. [46] presents a flank milling positioning methodology that is a modification of [44]. The author claims 88% less under-cutting then in [44]. The methodology consists of a 3-step optimization: Initialization of the position, making the tool tangent to a rule line, and finally making the tool tangential to two guiding rails (that are not the top/bottom curves) and one rule line. The algorithm efficiency was compared on the surface (Table 9) given in [21]. Results of this comparison are given in Table 2.

Chiou [47] suggests using the cutter swept envelope to compute the remaining machined part geometry. The swept envelope is considered as the totality of the points that belongs to the trace of the cutter. A methodology is proposed where the explicit swept profile of a taper-end cutter is computed. The author applies the proposition on a ruled surface (without referring to its equation). Moreover, no comparison between other methodologies with respect to error enhancement is provided. Weinert et al. [48] and Lee and Nestler [49] also proposed other swept volume generation methodologies.

Li et al. [45] studied three flank milling error measurements methods: The radial method [9,19,44,21], the parametric method and the closest point [30,31]. The authors concluded that the radial method seems poor – even though it is simple to compute – as it is not really calculating the error between the point on the ruled surface and the point on the machined surface and that the parametric method will fail to give an accurate measurement of the shortest distance between the ruled surface and the machined



(a) Positioning errors. (b) Optimized ruled surface.

Fig. 12. Original tool path errors on left for study given in [43], optimized ruled surface on right after application of proposition.

Table 3
Comparison of results shown in [45] for surface in Table 12 (units are in mm).

Method → Error ↓ (u)	Radial		Parametric		Motion	
0.2	−0.03	0.015	−0.025	0.04	−0.03	0.012
0.5	−0.041	0.04	−0.06	0.1	−0.03	0.015
0.9	−0.05	0.05	−0.12	0.17	−0.05	0.05

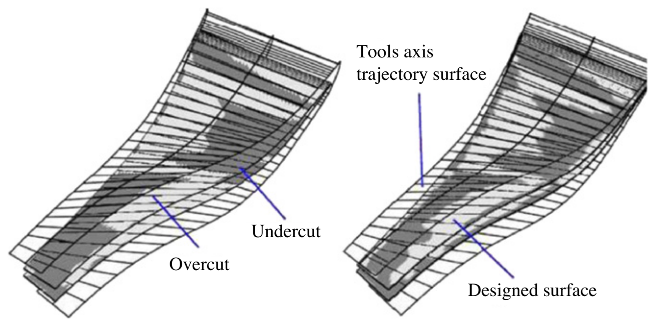


Fig. 13. Distribution of manufacturing errors for [50] (a) with TPO before optimization (b) with TPO after optimization.

Table 4
Machining deviations relevant to Fig. 13(a) (units are in mm).

	Maximum deviation	Average deviation	Standard deviation
Undercut	0.02084	0.00399	0.00329
Overcut	0.01932	0.00357	0.00286

surface. Following on from this, two methods are proposed: the tangent plane method and the motion method. In the first method (Tangent Plane), the tangent plane to the grazing surface is intersected with a line normal to the ruled surface. In the second method (Motion), the shorter distance between a ruled point and a line constructed from the point's corresponding grazing point in the direction motion is computed. Examples are carried on two test surfaces (points given in Tables 12 and 13). Results conclude that the proposed methodologies are simple and robust with better results (Table 3).

Gong et al. [50] uses a cylindrical tool to develop a positioning strategy to minimize error between the intended designed surface and the machined one. It is proved that errors are linked to the deviation between the tool axis trajectory surface and the offset surface of the designed surface. The initial tool trajectory is calculated using a proposed three-points offset method. Then, a least square method was used to fit the offset surfaces of the designed surface using tool trajectory surface. Results of the propositions are found in Fig. 13 and errors are reported in Tables 4 and 5.

Senatore et al. [5] criticizes efforts to deduce the optimal positioning using the envelope surface (in reference to [43]) and legitimizes the previous geometrical studies, mainly [26]. The authors further stress that [43] did not introduce the geometrical

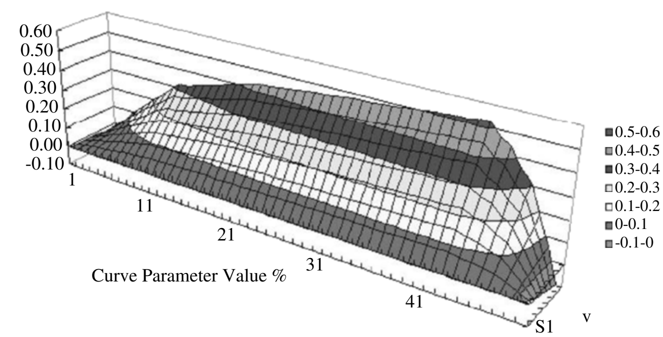


Fig. 14. Distribution of the approximation error applied on a test example of [51].

positioning used and that the surface treated is not twisted. Nonetheless, the article proposes a study of the error between the envelope and theoretical surfaces. An example with high values (larger cutter diameter and high twistability) showed accordance with the kinematic study (slight difference between real and calculated angles of tool rotation). Chu and Chen [51] presented a flank milling manufacturing approach where a general ruled surface is approximated with consecutive developable patches eliminating tool interference. The algorithm solves through a geometric computation while guaranteeing G1 continuity for tool path smoothness optimization. Fig. 14 shows the application test error results. The article does not present a smoothness parameter or formulation.

Senatore et al. [33] covered the analysis of the rotation axis influence on the generated errors. Besides the significant error enhancement of 10% upon optimization of the rotating axis, the article also offers a detailed identification of the error/radius error relations. In [34], a study to provide assistance in choosing the radius for milling cutters was proposed. The maximum errors (overcut–undercut) and the maximum radius are determined from an estimation of maximum error in relation to a tolerance interval. Gong et al. [52] presents a new methodology named BCELTP (Basic Curvature Equation of Locally Tool Positioning). The method relies on local optimization of tool position and unify the calculation of tool positions into a group equations for all kinds of cutting tools, which can handle both end milling and flank milling. This local applicability enables the user to adjust tool positions individually until the relative normal curvature between the envelope surface and the designed surface is minimized. The algorithm is applied and verified on several examples; Fig. 15 shows one of those examples relevant to flank milling. Sprott and Ravani [53] uses the geometric presentation of a surface to derive an offset method that intersects all of the normals along a ruling. The obtained line is constructed using the generator trihedron. The cutter is cylindrical. Test surface data are not given but rather a geometric figure.

Wu et al. [54] uses dynamic programming to solve the toolpath problem. Previous methodologies were classified as good for the local optimums, and that the assumption where the global optimum equals the sum of local optimums is not generally true.

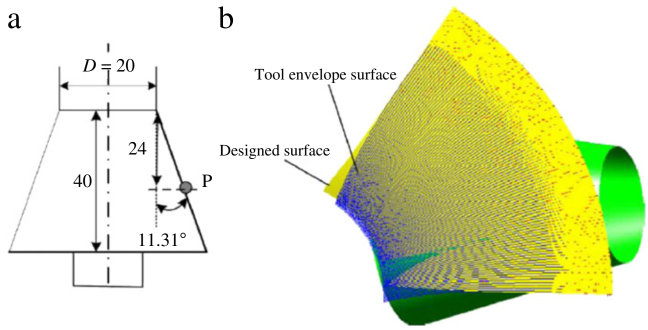


Fig. 15. Flank milling free form surfaces with conical cutter.

Table 5
Machining deviations relevant to Fig. 13(b) (units are in mm).

	Maximum deviation	Average deviation	Standard deviation
Undercut	0.01770	0.00252	0.00255
Overcut	0.01849	0.00213	0.00253

Table 6
Comparison of results shown in [55] for surface in Table 9 (units are in mm).

	[21]	[26]	[46]	[50]	[55]
Undercut	0.582	0.220	0.264	0.093	0.068
Overcut	0.585	0.220	0.211	0.119	0.067

The attempt is to transform the geometrical program of tool path generation into a mathematical programming problem. A Z-buffer is used for a quick estimation of the machining error. Ding and Zhu [55] proposes a global optimization methodology that is novel in terms of the perspective of approximating the tool envelope surface to the data points on the designed surface following the minimum zone criterion recommended by ANSI and ISO standards for tolerance evaluation. The interchangeability principle was developed to optimize the tool path. The method was compared with the main methodologies to date. Results are shown in Table 6.

Gong and Wang [4] presents an extension of [50] based on the proposed analytical calculation method of envelope surfaces using a moving frame [56]. It provides a new definition for the error computation, where it uses the latter to establish an optimization model to obtain the global optimized tool axis trajectory surface. The method is applied on the Archimedes' helicoid test surface of [5] (Table 14). The authors provided supplementary material to verify the algorithm in different CAD/CAM systems. They conclude on the universality of the methodology on both ruled and non-ruled surfaces as well as the usability of generic cutters. Pechard et al. [41] introduces the concept of energy optimization while generating the optimal toolpath. The target is to find a pseudo solution that gives minimal geometrical deviations while preserving a correct smoothness of the trajectory. The method, labeled Geo5XF, is applied on the surfaces of [46,5,21], as well as on an industrial "Impeller" surface. The results were compared in context (Tables 7 and 8). Zhu et al. [57] proposes a sphere congruence model to analytically compute the swept envelope surfaces of general rotary tools. With this model, an elegant approach was presented to efficiently compute the signed distance between a point in space and the swept surface without constructing the swept surface itself. The first-order differential increment of the signed point-to-surface distance with respect to the differential deformation of the tool axis trajectory surface was derived. It characterizes quantitatively the change of the geometric error of the machined surface under the change of the tool trajectory. The distance function theory is innovative

Table 7
Comparison of results on surface in [46] (units are in mm).

	[44]	[46]	[41]
Undercut	0.2876	0.0061	0.0086
Overcut	0	0.0091	0.016

Table 8
Comparison of results on surface in [5] (units are in mm).

	[21,26]	[50,5]	[46]
Undercut	0	0	0.007
Overcut	1.5	0.0165	0.07

and exciting. It provides a natural way to extend the approach presented in [55] to deal with generic rotary cutters [58,59]. Zheng et al. [60] recently used multi-objective programming: attempting to optimize simultaneously trajectory smoothness and machining errors. The tool path that yields the minimal geometric deviation was used to check the existence of the feasible solutions to the optimization problem. The proposed method was developed for a chosen cutter with a fixed type and size without consideration to global interference.

3.2. Summary map for toolpath generation

The review of tool trajectory optimization contains a handful of publications. To facilitate the reviewed concepts, Fig. 16 depicts the main publications:

- Early Works (1979–2001): Surveyed by Monies et al. [31] in 2001, it includes preliminary simple positioning by Stute et al. [19], the introduction of SPO & DPO methods by Liu [21], the investigation of dual positions by Rubio et al. [27], the enhancement of the latter by Redonnet et al. [26] as well as by Monies et al. [30].
- Optimization Work (2001–2011): Two main publications highlight this part: Lartigue et al. [43] and Bedi et al. [44]. The first introduced the concept of envelope surfaces on which [5,33,34] built different enhancements: Error computation between envelope and theoretical surfaces, optimization of rotating axis and linking the cutter radius to error. The second proposed tangential positioning and was enhanced by Menzel et al. [46] and Li and Jerard [22]. Other optimization techniques were introduced by Gong et al. [50]. The latter was extended in [4], and by Ding and Zhu [55].
- Novel Concepts (2008–2011): Recently, more novel concepts have been studied to manufacture non-developable surfaces outside of the regular optimization track. Wu et al. [54] introduced an attempt to use dynamic programming to solve the problem, Pechard et al. [41] energy minimization, Zhu et al. [57] tolerances and Gong and Wang [16] constraint surfaces.

4. Manufacturing tool adaptation

4.1. Chronological review

Studies involving manufacturing tools can be mainly categorized into: (1) Manufacturing tool geometrical change and adaptation in the calculation of the computed surface, and (2) tool chatter suppression and flank deflection generating β. Many studies have been carried out on chatter suppression in end or sweep milling—i.e. in [61–65], where tool cutting forces and wear were used for chatter suppression optimization, thus canceling β. Chou and Yan [66] investigates the importance of relating machine tool dynamics and control in a CAM process to the geometrical data

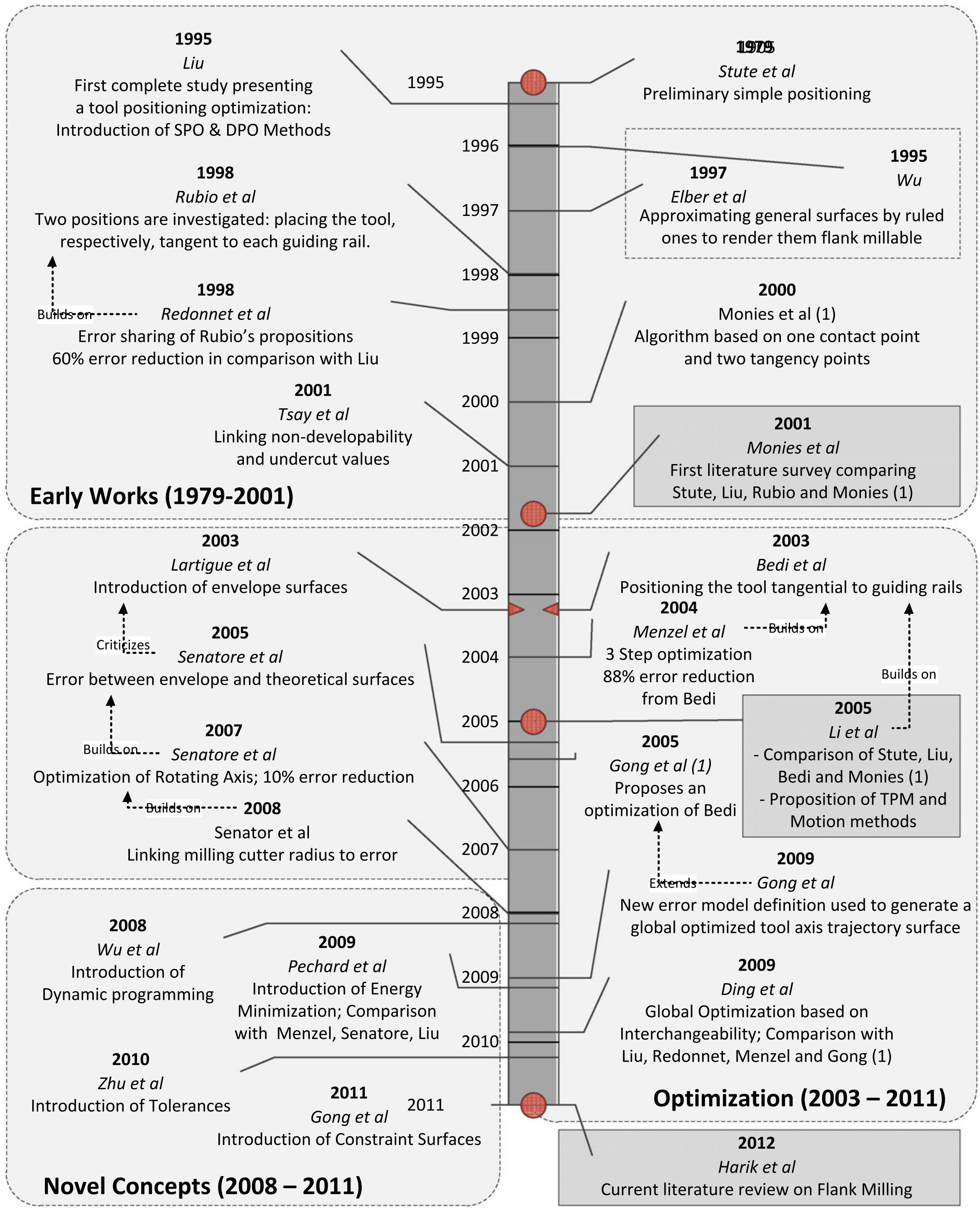


Fig. 16. Timeline of main flank milling toolpath generation references.

in a CAD model: Geometric properties of curves are related to dynamic properties of the coordinated cutting process. Ramaraj and Eleftheriou [67] presented a model for the mechanics of machining, using shear strain properties and variations of shear stress with applied normal stress. Wang et al. [68] presents a study of curve interproximation with different energy forms and parameterization techniques. The surface energy form might be used as an indicator of high non-developability. Erkorkmaz and Altintas [69] highlighted the important point that the generated trajectories must not only describe the desired tool path accurately, but must also have smooth kinematic profiles in order to maintain high tracking accuracy and avoid exciting the natural modes of the mechanical structure or servo control system. Monies et al. [32] examines optimal cutter selection based on [30,31]. It indicated that the choice of the cutter is a tradeoff: while small cutters produce greater theoretical accuracy, large cutters deflect less and remove material more quickly. Chiou and Lee [70] offers a comprehensive understanding of tool geometries as well as an enhanced *G*-Buffer model by the constructed swept envelope. Several techniques are offered to find the 3D shape-generating profiles. The studies do not involve flank milling, but rather end milling. Dugas et al. [71] presented a machining simulation for

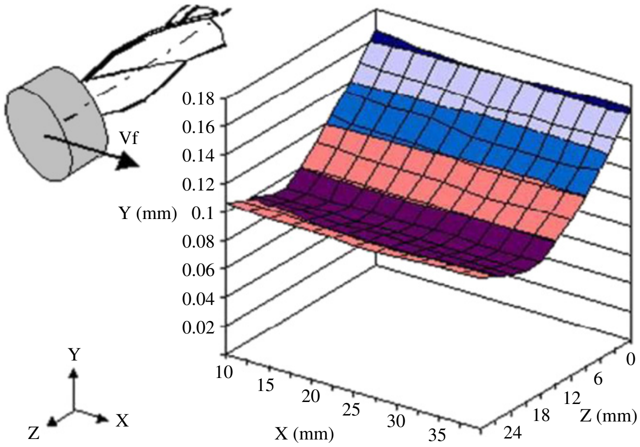


Fig. 17. Deformed surface of [74].

NC application. The simulation tool was developed locally at the IRCCyN laboratory. The main benefits of the tool are to analyze and localize errors obtained by tool deflection. The latter should be taken into account when computing the optimal tool trajectory. It would function as a corrective measure while manufacturing. The real feedrate and tool engagement are calculated, then the deflection is estimated and accounted for. Larue and Anselmetti [72] presented the deviation of a machined surface in flank milling due to tool deflection. The authors predicted the defects of the tool during cutting by applying an identification procedure of force model coefficients. The study was carried out on a planar surface and not a general ruled surface. The tool considered was assimilated to a fixed beam. The initial assumptions were further experimentally validated. The error between the theoretical calculated deflection and the digitally measured profile fell in a 0.1 mm tolerance. At a second stage, the author in [73] generalized the methodology and presented simulation of flank processes using multiple tool geometries and on a general ruled surface—instead of applying the force prediction model on a planar surface. The machining forces are predicted in the virtual environment by modeling the cutter/part intersection along the toolpath. For the purposes of this study, the authors used geometrical modeling to predict the forces. One of the main results would be the feedrate scheduling capacity of the model in such a way as to keep cutting forces at desirable ranges. Landon et al. [74] proves the possibility of predicting tool deflection with the use of a cutting force model. Models are experimentally driven. Fig. 17 shows the reconstruction of the deformed surface of a milled workpiece in side milling. Zhu et al. [58] provides a methodology to integrate the cutting force calculations in the generation of the toolpath. Two methodologies are presented distinctively for 3-axis and 5-axis machining. The cutting forces were calculated following a mechanistic model. Ferry and Yip-Hoi [75] and Ferry and Altintas [76,77] developed a complete understanding of 5-axis flank milling mechanics as well as a feedrate optimization technique. In [78], a predictive model integrating NC functions with axis capacities is presented. The formalism is applicable to a multitude of tool architectures. Quinsat et al. [79] proposed a method to characterize 3D topographies of a machined surface, integrating the effects of high velocities in the 5-axis machining setup. Chaves-Jacob et al. [80] suggested the adaptation of the tool geometry to eliminate the error. Instead of optimizing the tool path, a tool profile will be associated with the distance profiles between the tool axes and the surface at the preliminary toolpath (generated by a simple positioning).

4.2. Further considerations

Literature integrating optimal toolpath planning with force mechanistic models estimating deflections and variation on flank milling ruled surfaces has not been undertaken. Larue and Anselmetti [72] presents the most viable model to be further considered in undertaking the above integration. The model is based on the particular planar surface; however a generalization should be possible. The prediction model allowed the determination of the variations of the form and position of the surface points with a margin of 5%.

5. Other related works

5.1. Tool flank wear and temperatures

Analyzing cutting tool temperature [81,82] and measuring tool flank wear [83] provides additional insight – above tool deflection analysis – to understand the difference between the computed and machined surfaces. Several means of measuring the flank wear have been developed. Methods based on real time vision technology [84] are of particular interest for automated trajectory rectification. Estimating flank wear is of great concern since the amount of flank wear is often used in estimating the tool life [85].

Bhattacharyya and Ham [86] presented one of the first wear model analyses on the flank sides of the tool. These sides are the ones used to perform a flank milling operation. The model did not determine interaction coefficients but rather had them given, as well as the force data. The paper also presented a two-flank wear model split within two areas defined as the insensitive and sensitive regions. Karthik et al. [87] delivered a method to measure wear parameters to predict a potential breakage or failure of the tool. The parameters that are detected are area, average depth and volume of crater wear.

5.2. Sweep milling

Numerical control machining of free form surfaces studies was initiated using the more general sweep milling at first in [88], where interpolation techniques [89,90] were developed to ensure safe, concise and accurate machining of curved objects [91]. Studies aimed at improving the trajectories based on the geometric inputs (scallop heights and chordal deviation) rather than on abstract algebraic quantities. Some attempted to define the optimal tool geometry – i.e. [92] comparing ball-mills versus end-mills – while most concentrated on the optimal tool trajectory computation. Choi et al. [93] is amongst the first research published where the cutter location optimization problem is translated into a 2D constrained minimization problem. The algorithm computed the cutter location data, checked for gouging, computed joint values, checked the over-limit as well as the collision before submitting the optimal join values. If a set of cutter orientation angles is not found to be adequate, then new angles are proposed to be checked. The algorithm was applied to 5-axis face milling of marine propellers. You and Chu [94] presents a systematic scheme for the verification of tool paths in 5-axis machining of sculptured surfaces. The method is mainly applied to sweep milling and introduces an interesting concept of surface decomposition in order to estimate the error. The subdivision into discrete sample points allows a clear geometrical computation of the error. The methodology verification is presented and demonstrated. Even though the application is in sweep milling, the methodology of error detection might still be applicable for flank milling. Another sweep milling methodology based on the envelope differential equation to characterize the tool swept volume is found in [95]. The algorithm uses tool grazing points to reduce computation complexity. Warkentin et al. [96] proposes an intersection approach to multi-point machining of sculptured surfaces. The authors present the methodology using a ball nose tool performing sweep milling. The concept is to have the tool in contact with the surface at multiple locations (the tip of the tool). The scallop left by the tool path is to be minimized. The work presents a list of commonly used cutting tools as well as the multi-point contact between a toroidal cutter and a surface. Results are computed through geometrical transformations and calculations. There is an abundant set of examples, mainly one considering machining a cubic Bezier surface. Duc et al. [40] presents a concept of a 'machining surface' (not to be confused with our definition) that embeds all toolpath information. The application of the 'machining surface' ensures a good concordance between the first design intents and the machined part. Rao and Sarma [97] suggests a method for the removal of excess material in the vicinity of the cutter contact point due to the mismatch in curvatures between the tool and the designed surface. This article applies to sweep milling where tool is considered a flat-end tool. The method finds the curvatures of the tool swept surface and then gouging is detected by comparing it with the designed surface. Roth et al. [98] presents a method to determine the imprint curve of tools as it executes a 5-axis tool movement. It is applied on sweep milling. The obtained surface can be used to rectify the tool trajectory to reduce errors. The method, assessed by two tests, makes the approximation by assuming piecewise linear motion of the tool. Roth et al. [99] complements [98] by proposing a volumetric force model of the milling process based on the depth buffer of a high end rendering engine. The method allows for increased simulation accuracy while reducing memory requirements. It allows force magnitudes and variations of a full 3-axis cut to be determined. Chiou and Lee [100] presents a closed-form solution of the swept profile of a generalized cutter in 5-axis NC machining. The approach is of interest because it is based on the machine configurations as well as the exact tool movements recorded in NC part programs. The authors had previously presented [101] another approach to generate 5-axis tool paths, called machining potential field. The latter searches for a preliminary optimal tool path of which adjacent toolpaths are proposed. Fussell et al. [102] uses a Z-buffer approach to model the part geometry during a 5-axis sculptured surface machining. The application is concerned with end milling and the intersection between the tool path and part, which is evaluated using a 3-axis approximation. Experimentation showed that the model is accurate except under large transient conditions. Yoon et al. [103] and Ho et al. [104], respectively, present sweep milling optimization and trajectory smoothing algorithms. The latter [104] show that error computations are sometimes computed linearly whereas, due to tool rotation, the real cutting errors are higher than theoretical values. Affouard et al. [105] presented a topical method for avoiding the tool traversing singular positions in 5-axis sweep milling. The approach deforms the tool path until the maximum displacement of the tool is enough to leave the singular cone. This will also improve the performance dynamics. Langeron et al. [106] generated a new format of tool path polynomial interpolation in 5-axis machining using bspline curves to reduce the usually produced tangency discontinuities. Lamikiz et al. [107] presented a methodology of estimating the precision of complex 5-axis milling centers based on the Denavit and Hartenberg formulation. Even though the work was applied to multi-axis drilling rather than our subject of interest – flank milling – it clearly shows the importance as well as the complexity of kinematics when manipulating 5-axis manufacturing operations. The methodology proposed estimating assembly errors and then reducing those with more weight on the tool tip error.

5.3. Further considerations

In [72], 5% of the unpredicted tool deviation was attributed to a phenomenon not yet identified. We believe that further investigation into tool wear and cutting temperatures might optimize and further reduce this error. Moreover, we have covered sweep milling, mainly the references that might be projected for our needs, i.e. error verification and swept volume generation.

6. Summary

In the following section we present a summary of our findings surveying flank milling literature. We also gather all the test surfaces for future comparisons.

6.1. Conclusions

We can draw the following conclusions from our review of the literature:

(1) Most of the papers focus on the development of optimal tool path of flank milling, the objective of which is to minimize the geometrical machining errors between the tool envelope surfaces and the designed surface. Great improvement has been achieved based on academic and industrial colleges' efforts.
(2) Flank milling technology is mostly used on ruled surfaces, because cutting tools are cylindrical surfaces or conical surfaces, which are all ruled surfaces. Moreover, some methods have also tried to flank mill general free-form surfaces, which are close to ruled surfaces. Currently, machining errors from toolpath planning may be negligible for these surfaces.
(3) The basic objective of flank milling is to improve machining strip width and reduce machining error simultaneously. More than ruled surfaces or quasi ruled surfaces, we may extend this idea to more general free-form surfaces. Our concern is how to determine whether these surfaces can be machined using flank milling technology. It will be meaningful to extend them to more general cutting tools for more general surfaces to improve machining efficiency.
(4) Compared with end milling, flank milling has higher cutting forces, which may lead to more deformation of cutting tools and generate extra errors, even the roughness get worse. Apart from the cutting force, there are some other relevant factors, such as tool geometry and the parameters of flank milling. In future work, more attention may be paid to the design of cutting tools for flank milling and optimization of the machining parameters according to the cutting force or roughness of machined surface.
(5) The issue of geometrical error has been discussed widely. But this is only calculated in the workpiece coordinate system. If we transform the toolpath into the motions of each axis respectively, extra kinematics error will be introduced. In addition, dynamics issue will be raised too. Therefore, toolpath generation incorporating consideration of post-processing may be paid more attention to in future research.
(6) Flank milling of planar surfaces has not been treated. Geometrically, flank milling planar surfaces does not represent any challenge as any selected tool direction is optimal (error is null). The complexity remains in the toolpath energy minimization and optimal boundary manufacturing combination.

6.2. Test surfaces

Tables 9–14 constitute the test surfaces used in several examples. Future studies aiming at studying flank milling should provide results on the surfaces below in comparison with other methodologies.

Table 9
Test surface of [21].

	T_0	T_1	T_2	B_0	B_1	B_2
x	0	11.507	23.014	0	11.507	23.014
y	0	0	20.2324	20.429	20.429	20.429
z	33.995	33.995	33.995	0	0	0

Table 10
Test surface of [26].

$$S(u,v) = \begin{cases} 30u^2 + 10u + 30 + 20vu^2 - 10vu - 10v \\ 100\sqrt{u} - 50 - 100v\sqrt{u} + 90v - 60v\sin\left(\frac{1}{2}\pi u\right) \\ 70 - 30u^2 - 130v + 50vu^2 \end{cases}$$

Table 11
Test surface of [44].

	T_0	T_1	T_2	B_0	B_1	B_2
x	1	0	0	1	−0.25	0
y	0	0	1	0	−0.25	1
z	1	1	1	0	−1	0

Table 12
First test surface of [45].

	T_0	T_1	T_2	B_0	B_1	B_2
x	65	30	0	60	30	15
y	15	30	60	9	30	75
z	−5	−5	−5	−35	−35	−35

Table 13
Second test surface of [45].

	T_0	T_1	T_2	T_3	B_0	B_1	B_2	B_3
x	5	40	75	110	5	40	75	110
y	65	35	35	65	50	80	20	50
z	−35	−35	−35	−35	−5	−5	−5	−5

Table 14
Test surface of [5].

$$S(u,v) = \begin{cases} 20(2v-1)\cos\left(\frac{\pi}{4}(1-2u)\right) \\ 37.5(2u-1) \\ 20(2v-1)\sin\left(\frac{\pi}{4}(1-2u)\right) \end{cases}$$

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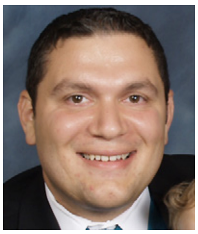
References

[1] Young HT, Chuang LC, Gerschwiler K, Kamps S. A five-axis rough machining approach for a centrifugal impeller. International Journal of Advanced Manufacturing Technology 2004;23:233–9.
[2] Tönshoff H, Gey C, Rackow N. Flank milling optimization—the Flamingo project. Air and Space Europe 2001;3:60–3.
[3] Peternell M, Pottmann H, Ravani B. On the computational geometry of ruled surfaces. Computer-Aided Design 1999;31:17–32.
[4] Gong H, Wang N. Optimize tool path of flank milling with generic cutter based on approximation using envelope surface. Computer-Aided Design 2009; 41(12):981–9.
[5] Senatore J, Monies F, Redonnet JM, Rubio W. Analysis of improved positioning in five-axis ruled surface milling using envelope surface. Computer-Aided Design 2005;37(10):989–98.
[6] Pottmann H, Wallner J. Approximation algorithms for developable surfaces. Computer-Aided Design 2005;3:1459–68.
[7] Chen HY, Pottmann H. Approximation by ruled surfaces. Journal of computational and Applied Mathematics 1999;102:143–56.
[8] Bohez E. Compensating for systematic errors in 5-axis NC machining. Computer-Aided Design 2001;34:391–403.
[9] Bohez E, Ranjith Senadhera SD, Pole K, Duflou JR, Tar T. A geometric modeling and five-axis machining algorithm for centrifugal impellers. Journal of Manufacturing Systems 1997;16(6):422–36.
[10] Bohez E, Makhanov SS, Sonthipermpoon K. Adaptive nonlinear tool path optimization for five-axis machining. International Journal of Production Research 2000;38(17):4329–43.
[11] Bohez E. Five-axis milling machine tool kinematic chain design and analysis. International Journal of Machine Tools and Manufacture 2002;42:505–20.
[12] Suh S, Lee K. Avoiding tool interference in four-axis NC machining of rotationally-free-surfaces. IEEE Transactions on Robotics & Automation 1992;v8:718–29.
[13] Takeuchi Y, Watanabe T. Generation of 5-axis control collision free tool path planning and postprocessing for NC data. Annals of the CIRP 1992;41: 539–42.
[14] Lee YS. Admissible tool orientation control of gouging avoidance for 5-axis complex surface machining. Computer-Aided Design 1997;29(7):507–21.
[15] Lee YS. Non-isoparametric tool path planning by machining stip evaluation for 5-axis sculptured surface machining. Computer-Aided Design 1998; 30(7):559–70.
[16] Gong H, Wang N. 5-axis flank milling free-form surfaces considering constraints. Computer-Aided Design 2011;43(6):563–72.
[17] Zheng G, Zhu LM, Bi QZ. Cutter size optimisation and interference-free tool path generation for five-axis flank milling of centrifugal impellers. International Journal of Production Research. 2011. on line. http://dx.doi.org/10.1080/00207543.2011.611631.
[18] Castagnetti C, Duc E, Ray P. The domain of admissible orientation concept: a new method for five-axis tool path optimisation. Computer-Aided Design 2008;40:938–50.
[19] Stute G, Storr A, Sielaff W. NC programming of ruled surfaces for five axis machining. Annals of the CIRP 1979;28(1):267–71.
[20] Rehsteiner F, Rewker HJ. Collision free five axis milling of twisted ruled surface. Annals of the CIRP 1993;42(1):457–61.
[21] Liu XW. Five-axis NC cylindrical milling of sculptured surfaces. Computer-Aided Design 1995;27(12):87–94.
[22] Li SX, Jerard RB. Five-axis machining of sculptured surfaces with a flat-end cutter. Computer-Aided Design 1994;26(3):165–78.
[23] Chappel IT. The use of vectors to simulate material removed by numerically controlled milling. Computer-Aided Design 1983;15(3):156–8.
[24] Wu CY. Arbitrary surface flank milling of fan, compressor and impeller blades, transactions of the ASME. Journal of Engineering for Gas Turbines and Power 1995;117:534–9.
[25] Elber G, Fish R. 5-Axis freeform surface milling using piecewise ruled surface approximation. Journal of Manufacturing Science and Engineering 1997;119: 383–7.
[26] Redonnet J, Rubio W, Dessein G. Side milling of ruled surfaces; optimum positioning of the milling cutter and calculation of interference. The International Journal of Advanced Manufacturing Technology 1998;14(7): 459–65.
[27] Rubio W, Lagarrigue P, Dessein G, Pastor F. Calculation of tool paths for a torus mill on freeform surfaces on five-axis machines with detection and elimination of interface. International Journal of Advanced Manufacturing Technology 1998;14:13–20.
[28] Abdel-Malek K, Yeh HJ. Geometric representation of the swept volume using Jacobian rank-deficiency conditions. Computer-Aided Design 1997;29(6): 457–68.
[29] Leu MC, Wang L, Blackmore D. A verification program for 5-axis machining with general APT tools. Annals of the CIRP 1997;46(1):419–24.
[30] Monies F, Redonnet JM, Rubio W, Lagarrigue P. Improved positioning of a conical mill for machining ruled surfaces. Journal of Engineering Manufacture 2000;214(7):625–34.
[31] Monies F, Rubio W, Redonnet JM, Lagarrigue P. Comparative study of interference caused by different position settings of a conical milling cutter on a ruled surface. Journal of Engineering Manufacture 2001;215(B9): 1305–17.
[32] Monies F, Felices JN, Rubio W, Redonnet JM, Lagarrigue P. Five axis NC milling of ruled surfaces: optimal geometry of a conical tool. International Journal of Production Research 2002;40(12):2901–22.
[33] Senatore J, Monies F, Redonnet JM, Rubio W. Improved positioning for side milling of ruled surface: analysis of the rotation axis's influence on machining error. International Journal of Machine Tools and Manufacture 2007;47: 934–45.
[34] Senatore J, Landon Y, Rubio W. Analytical estimation of error in flank milling of ruled surfaces. Computer-Aided Design 2008;40:595–603.
[35] Lin RS, Koren Y. Ruled surface machining on five-axis CNC machine tools. Journal of Manufacturing Processes 2000;1(2):25–35.
[36] Tsay DM, Her MJ. Accurate 5-axis machining of twisted ruled surfaces. ASME Journal of Manufacturing Science and Engineering 2001;123:731–8.
[37] Tsay DM, Chen HC, Her MJ. A study on five-axis flank machining of centrifugal compressor impellers. ASME Journal of Engineering for Gas Turbines and Power 2002;124:177–81.
[38] Kim JH, Ryuh BS, Pennock GR. Development of a trajectory generation method for a five-axis NC machine. Mechanism and Machine Theory 2001; 36:983–96.

[39] Tournier C, Duc E, Lartigue C, Contri A. The concept of the machining surface in 5-axis milling of free form surfaces. In: Integrated design and manufacturing in mechanical engineering. Kluwer Academic Publishers; 2002. p. 279–86.
[40] Duc E, Lartigue C, Tournier C, Bourdet P. A new concept for the design and the manufacturing of free-form surfaces: the machining surface. Annals of the CIRP 1999;48(1):103–6.
[41] Pechard PY, Tournier C, Lartigue C, Lugarini JP. Geometrical deviations versus smoothness in 5-axis high-speed flank milling. International Journal of Machine Tools and Manufacture 2009;49:453–61.
[42] Beudaert X, Pechard PY, Tournier C. 5-Axis tool path smoothing based on drive constraints. International Journal of Machine Tools & Manufacture 2011;51:958–65.
[43] Lartigue C, Duc E, Affouard A. Tool path deformation in 5-axis flank milling using envelope surface. Computer-Aided Design 2003;35:375–82.
[44] Bedi S, Mann S, Menzel C. Flank milling with flat end milling cutters. Computer-Aided Design 2003;35:293–300.
[45] Li C, Mann S, Bedi S. Error measurements for flank milling. Computer-Aided Design 2005;37(14):1459–68.
[46] Menzel C, Bedi S, Mann S. Triple tangent flank milling of ruled surfaces. Computer-Aided Design 2004;36(3):296–389.
[47] Chiou JCJ. Accurate tool position for five-axis ruled surface machining by swept envelope approach. Computer-Aided Design 2004;36(10):967–74.
[48] Weinert K, Du S, Damm P, Stautner M. Swept volume generation for the simulation of machining processes. International Journal of Machine Tools & Manufacture 2004;44:617–28.
[49] Lee SW, Nestler A. Complete swept volume generation, part I: swept volume of a piecewise C1-continuous cutter at five-axis milling via Gauss map. Computer-Aided Design 2011;43(4):427–41.
[50] Gong H, Cao LX, Liu J. Improved positioning of cylindrical cutter for flank milling ruled surfaces. Computer-Aided Design 2005;37(12):1205–13.
[51] Chu CH, Chen JT. Tool path planning for five-axis flank milling with developable surface approximation. International Journal of Advanced Manufacturing Technology 2006;29(7–8):707–13.
[52] Gong H, Cao LX, Liu J. Second order approximation of tool envelope surface for 5-axis machining with single point contact. Computer-Aided Design 2008; 40(5):604–15.
[53] Sprott K, Ravani B. Cylindrical milling of ruled surfaces. International Journal of Advanced Manufacturing Technology 2008;38:649–56.
[54] Wu PH, Li YW, Chu CH. Optimized tool path generation based on dynamic programming for five-axis flank milling of rule surface. International Journal of Machine Tools and Manufacture 2008;48:1224–33.
[55] Ding H, Zhu LM. Global optimization of tool path for five-axis flank milling with a cylindrical cutter. Science in China Series E-Technological Sciences 2009;52(8):2449–59.
[56] Gong H, Wang N. Analytical calculation of envelope surface for generic milling tool directly from CL-data based on moving frame. Computer-Aided Design 2009;41(11):848–55.
[57] Zhu LM, Zhang XM, Zheng G, Ding H. Analytical expression of the swept surface of a rotary cutter using the envelope theory of sphere congruence. Journal of Manufacturing Science and Engineering-Transactions of the ASME 2009;131(4):041017.
[58] Zhu LM, Zheng G, Ding H, Xiong YL. Global optimization of tool path for five-axis flank milling with conical cutter. Computer-Aided Design 2010;42(10): 903–10.
[59] Zhang XM, Zhu LM, Zheng G, Ding H. Tool path optimisation for flank milling ruled surface based on the distance function. International Journal of Production Research 2010;48(14):4233–51.
[60] Zheng G, Bi QZ, Zhu LM. Smooth tool path generation for five-axis flank milling using multi-objective programming. Proceedings of the Institution of Mechanical Engineers Part B-Journal of Engineering Manufacture 2012; 226(2):247–54.
[61] Abrari F, Elbestawi MA, Spence AD. On the dynamics of ball end milling: modeling of cutting forces and stability analysis. International Journal of Machine Tools and Manufacture 1998;38:215–37.
[62] Ismail F, Elbestawi MA, Du R, Urbasik K. Generation of milled surfaces including tool dynamics and wear. Journal of Engineering for Industry 1993; 115:245–52.
[63] Ismail F, Ziaei R. Chatter suppression in five-axis machining of flexible parts. International Journal of Machine Tools & Manufacture 2002;42:115–22.
[64] Tlusty J, Ismail F. Basic nonlinearity in machining chatter. Annals of the CIRP 1981;299–304.
[65] Elbestawi MA, Ismail F, Du R, Ullagaddi BC. Modelling machining dynamics including damping in the tool workpiece interface. Journal of Engineering for Industry 1994;116:435–9.
[66] Chou JJ, Yan D. Command generation for three-axis CNC machining. ASME Journal of Engineering Industry 1991;113:305–10.
[67] Ramaraj TC, Eleftheriou E. Analysis of the mechanics of machining with tapered end milling cutters. Transactions of ASME 1994;116:398–404.
[68] Wang X, Cheng F, Barsky B. Energy and *B*-spline interproximation. Computer-Aided Design 1997;29(7):485–96.
[69] Erkorkmaz K, Altintas Y. High speed CNC system design, part I: jerk limited trajectory generation and quintic spline interpolation. International Journal of Machine Tools & Manufacture 2001;41:1323–45.
[70] Chiou JCJ, Lee YS. A shape-generating approach for multi-axis machining *G*-buffer models. Computer-Aided Design 1999;31:761–76.
[71] Dugas A, Lee J, Hascoet JY. An enhanced machining simulator with error analysis. Journal of Manufacturing Systems 2003;21(6):451–564.
[72] Larue A, Anselmetti B. Deviation of a machined surface in flank milling. International Journal of Machine Tools and Manufacture 2003;43(2): 129–39.
[73] Larue A, Altintas Y. Simulation of flank milling process. International Journal of Machine Tools and Manufacture 2005;45:549–59.
[74] Landon Y, Segonds S, Lascoumes P, Lagarrigue P. Tool positioning error (TPE) characterisation in milling. International Journal of Machine Tools and Manufacture 2004;44(5):457–64.
[75] Ferry WB, Yip-Hoi D. Cutter-workpiece engagement calculations by parallel slicing for five-axis flank milling of jet engine impellers. ASME Journal of Manufacturing Science and Engineering 2008;130.
[76] Ferry WB, Altintas Y. Virtual five-axis flank milling of jet engine impellers—part I: mechanics of five-axis flank milling. Journal of Manufacturing Science and Engineering 2008;130.
[77] Ferry WB, Altintas Y. Virtual five-axis flank milling of jet engine impellers—part II: feed rate optimization of five-axis flank milling. Journal of Manufacturing Science and Engineering 2008;130.
[78] Lavernhe S, Tournier C, Lartigue C. Kinematical performance prediction in multi-axis machining for process planning optimization. International Journal of Advanced Manufacturing Technology 2008;37(5–6):534–44.
[79] Quinsat Y, Lavernhe S, Lartigue C. Characterization of 3D surface topography in 5-axis milling. Wear 2011;271(3–4):590–5.
[80] Chaves-Jacob J, Poulachon G, Emmanuel D. New approach to 5-axis flank milling of free-form surfaces: computation of adapted tool shape. Computer-Aided Design 2009;41:918–29.
[81] Loewen EG, Shaw MC. On the analysis of cutting-tool temperatures. Transactions of the ASME 1954;76:217–31.
[82] Yellowley I, Barrow G. The influence of thermal cycling on tool life in peripheral milling. International Journal of Machine Tools and Manufacture 1976;16:1–12.
[83] Giardini C, Ceretti E, Maccarini G. A neural network architecture for tool wear detection through digital camera observations. The International Journal of Advanced Manufacturing Technology 1996;372:137–44.
[84] Jeon JU, Kim SW. Optical flank wear monitoring of cutting tools by image processing. Wear 1988;127:207–17.
[85] Park JJ, Ulsoy AG. On-line flank wear estimation using an adaptive observer and computer vision, part 2: experiment. Transactions of the ASME Journal of Engineering for Industry 1993;115:30–43.
[86] Bhattacharyya A, Ham I. Analysis of tool wear-part 1: theoretical model of flank wear. ASME Journal of Engineering for Industry 1969;790–8.
[87] Karthik A, Chandra S, Ramamoorthy B, Das S. 3D tool wear measurement and visualization using stereo imaging. International Journal of Machine Tools and Manufacture 1997;37(11):1573–81.
[88] Loney G, Ozsoy T. NC machining of free form surfaces. Computer-Aided Design 1987;19:85–90.
[89] Shpitalni M, Koren Y, Lo CC. Real time curve interpolators. Computer-Aided Design 1994;26(11):832–8.
[90] Koren Y, Lin RS. Five-axis surface interpolators. Annals of the CIRP 1995;44: 379–82.
[91] Bedi S, Ali I, Quan N. Advanced interpolation techniques for CNC machines. ASME Journal of Engineering Industry 1993;115:329–36.
[92] Vickers GB, Guan KW. Ball-mills versus end-mills for curved surface machining. ASME Journal of Engineering for Industry 1989;111:22–6.
[93] Choi BK, Park JW, Jun CS. Cutter-location data optimization in 5-axis surface machining. Computer-Aided Design 1993;25(6):377–86.
[94] You CF, Chu CH. Tool-path verification in five-axis machining of sculptured surfaces. International Journal of Advanced Manufacturing Technology 1997; 13(4):248–55.
[95] Blackmore D, Leu MC, Wang LP. The sweep-envelope differential equation algorithm and its application to NC machining verification. Computer-Aided Design 1997;29(9):629–37.
[96] Warkentin A, Ismail F, Bedi S. Intersection approach to multi-point machining of sculptured surfaces. Computer Aided Geometric Design 1998;15: 567–84.
[97] Rao A, Sarma R. On local gouging in five-axis sculptured surface machining using flat-end tools. Computer-Aided Design 2000;32:409–20.
[98] Roth D, Bedi S, Ismail F, Mann S. Surface swept by a toroidal cutter during 5-axis machining. Computer-Aided Design 2001;33(1):57–63.
[99] Roth D, Ismail F, Bedi S. Mechanistic modeling of the milling process using an adaptive depth buffer. Computer-Aided Design 2003;35:1287–303.
[100] Chiou JCJ, Lee YS. Swept surface determination for 5-axis numerical control machining. International Journal of Machine Tools and Manufacture 2002; 42:1497–507.
[101] Chiou JCJ, Lee YS. A machining potential field approach to tool path generation for multi-axis sculptured surface machining. Computer-Aided Design 2002;34:357–71.
[102] Fussell BK, Jerard RB, Hemmett JG. Modeling of cutting geometry and forces for 5-axis sculptured surface machining. Computer-Aided Design 2003;35: 333–46.
[103] Yoon JH, Pottmann H, Lee YS. Locally optimal cutting positions for 5-axis sculptured surface machining. Computer-Aided Design 2003;35:69–81.
[104] Ho MC, Hwang YR, Hu CH. Five-axis tool orientation smoothing using quaternion interpolation algorithm. International Journal of Machine Tools and Manufacture 2003;43(12):1259–67.

[105] Affouard A, Duc E, Lartigue C, Langeron JM, Bourdet P. Avoiding 5-axis singularities using tool path deformation. International Journal of Machine Tools and Manufacture 2004;44(4):415–25.
[106] Langeron JM, Duc E, Lartigue C, Bourdet P. A new format for 5-axis tool path computation, using bspline curves. Computer-Aided Design 2004;36: 1219–29.
[107] Lamikiz A, López de Lacalle LN, Ocerin O, Díez D, Maidagan E. The Denavit and Hartenberg approach applied to evaluate the consequences in the tool tip position of geometrical errors in five-axis milling centres. Journal of Advanced Manufacturing Technology 2008;37:122–39.



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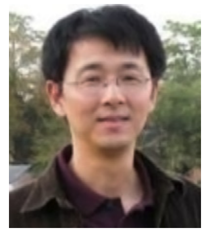

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